Assessment of SAR Image Filtering using Adaptive Stack Filters

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Abstract

Stack filters are a special case of non-linear filters. They have a good performance for filtering images with different types of noise while preserving edges and details. A stack filter decomposes an input image into several binary images according to a set of thresholds. Each binary image is then filtered by a Boolean function, which characterizes the filter. Adaptive stack filters can be designed to be optimal; they are computed from a pair of images consisting of an ideal noiseless image and its noisy version. In this work we study the performance of adaptive stack filters when they are applied to Synthetic Aperture Radar (SAR) images. This is done by evaluating the quality of the filtered images through the use of suitable image quality indexes and by measuring the classification accuracy of the resulting images.

The Multiplicative Model

Figure: Speckle, non aditive and non gaussiann noise. The noise is multiplicative and $\mathcal{G}^0$ distributed. Real image, Munich city

$$\mathbf{Z} = \mathbf{X} \cdot \mathbf{Y}$$

Where: $\mathbf{Z}$ is *Return*, $\mathbf{X}$ is *Backscatter* and $\mathbf{Y}$ is *Speckle Noise*

$\mathbf{X}$, $\mathbf{Y}$ independent random variables.

The intensity $\mathcal{G}^0$ distribution that describes speckled return is characterized by the following density:

$f(z) = \frac{L^L \Gamma(L-\alpha)}{\gamma^\alpha \Gamma(L)\Gamma(-\alpha)} \frac{z^{L-1}}{(\gamma + Lz)^{L-\alpha}}$, where $-\alpha, \gamma, z > 0$, $L \geq 1$, denoted $\mathcal{G}^0(\alpha, \gamma, L)$.

Many filters have been proposed in the literature for combating speckle noise, among them the ones by Lee and by Frost.

Our proposal: Stack Filter

The threshold is the set of operators $T^m \colon \{0, \ldots, M\} \to \{0, 1\}$ given by

$$T^m(x) = \begin{cases} 1 \text{ if } x \geq m, \\ 0 \text{ if } x < m. \end{cases}$$

A *boolean function* $f \colon \{0,1\}^n \to \{0,1\}$, where n is the length of the input vectors, has the stacking property if and only if

$$\forall X, Y \in \{0,1\}^n,\ X \leq Y \Rightarrow f(X) \leq f(Y).$$

We say that f is a *positive boolean function* if and only if it can be written by means of an expression that contains only non-complemented input variables. That is,

$$f(x_1, x_2, \ldots, x_n) = \bigvee_{i=1}^{K} \bigwedge_{j \in P_i} x_j, \qquad (1)$$

where n is the number of arguments of the function, K is the number of terms of the expression and P_i is a subset of the interval $\{1, \ldots, N\}$. '$\bigvee$' and '$\bigwedge$' are the AND and OR Boolean operators. It is possible to proof that this type of functions has the stacking property.

A stack filter is defined by the function $S_f \colon \{0, \ldots, M\}^n \to \{0, \ldots, M\}$, corresponding to the Positive Boolean function $f(x_1, x_2, \ldots, x_n)$ expressed in (1). The function S_f can be expressed by means of

$$S_f(X) = \sum_{m=1}^{M} f(T^m(X)).$$

Stack filters are built by a training process that generates a positive boolean function that preserves the stacking property. See [1, 2, 4]

References

- J. Astola and P. Kuosmanen. *Fundamentals of Nonlinear Digital Filtering*. CRC Press, Boca Raton, 1997.
- J.-H. Lin and Y. T. Kim. Fast algorithms for training stack filters. *IEEE Transactions on Signal Processing*, 42(3):772–781, 4 1994.
- Z. Wang and A.C. Bovik. A universal image quality index. *Signal Processing Letters, IEEE*, 9(3):81 –84, mar 2002.
- J. Yoo, K. L. Fong, J.-J. Huang, E. J. Coyle, and G. B. Adams III. A fast algorithm for designing stack filters. *IEEE Transactions on Image Processing*, 8(8):772–781, 8 1999.

Quality Index

We use the universal image quality index [3] and the correlation measure β. The universal image quality index $\mathbf{Q}$ is given by equation (2)

$$Q = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} \frac{2\overline{X}\overline{Y}}{\overline{X}^2 + \overline{Y}^2} \frac{2\sigma_X \sigma_Y}{\sigma_X^2 + \sigma_Y^2}, \qquad (2)$$

where $\sigma_X^2 = (N-1)^{-1}\Sigma_{i=1}^N (X_i - \overline{X})^2$, $\sigma_Y^2 = (N-1)^{-1}\Sigma_{i=1}^N (Y_i - \overline{Y})^2$, $\overline{X} = N^{-1}\Sigma_{i=1}^N X_i$ and $\overline{Y} = N^{-1}\Sigma_{i=1}^N Y_i$. The dynamic range of index $\mathbf{Q}$ is $[-1, 1]$, being $\mathbf{1}$ the best value.

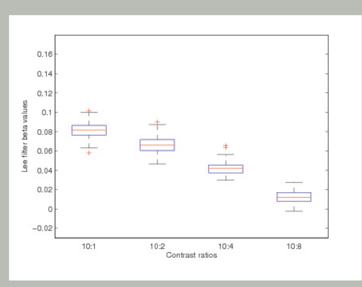
The correlation measure is given by

$$\beta = \frac{\sigma_{\nabla^2 X \nabla^2 Y}}{\sigma^2_{\nabla^2 X} \sigma^2_{\nabla^2 Y}}, \qquad (3)$$

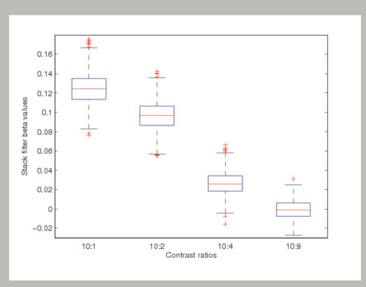
where $\nabla^2 X$ and $\nabla^2 Y$ are the Laplacians of images $\mathbf{X}$ and $\mathbf{Y}$, respectively. The correlation measure range is $[-1, 1]$.

Table: Statistics from image quality indexes

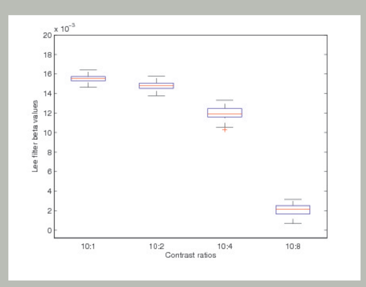
	β index				Q index			
	Stack filter		Lee filter		Stack filter		Lee filter	
contrast	$\overline{\beta}$	s_β	$\overline{\beta}$	s_β	$\overline{Q}$	s_Q	$\overline{Q}$	s_Q
10:1	0.1245	0.0156	0.0833	0.0086	0.0159	0.0005	0.0156	0.0004
10:2	0.0964	0.0151	0.0663	0.0079	0.0154	0.0005	0.0148	0.0004
10:4	0.0267	0.0119	0.0421	0.0064	0.0124	0.0008	0.0120	0.0006
10:8	−0.0008	0.0099	0.0124	0.0064	0.0041	0.0013	0.0021	0.0006



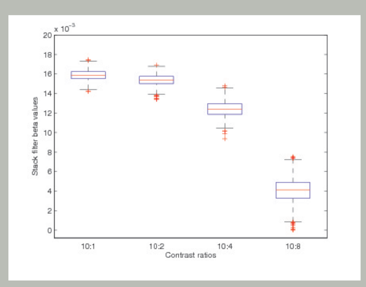
(a) Values of β, Lee filter



(b) Values of β, Stack filter



(c) Values of $\mathbf{Q}$, Lee filter

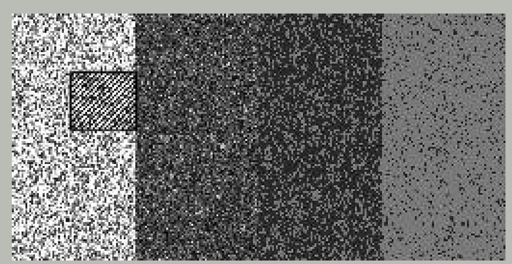


(d) Values of $\mathbf{Q}$, Stack filter

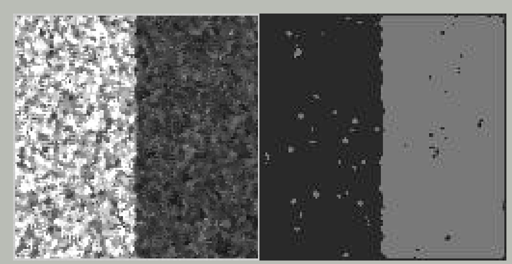
the image resulting of applyng the stack filter 95 times and the image produced by the application of the Lee filter. The presented results are the mean values obtained from a Monte Carlo experiment involving different contrast ratios.

Quality Classification

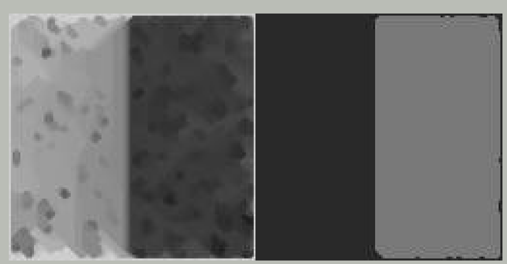
The quality of the classification results are obtained by calculating the confusion matrix, after Gaussian Maximum Likelihood Classification (GMLC).



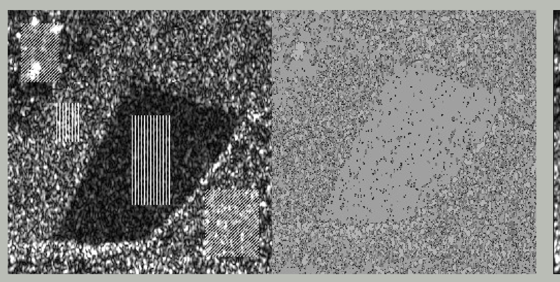
(e) Simulated image and GMLC



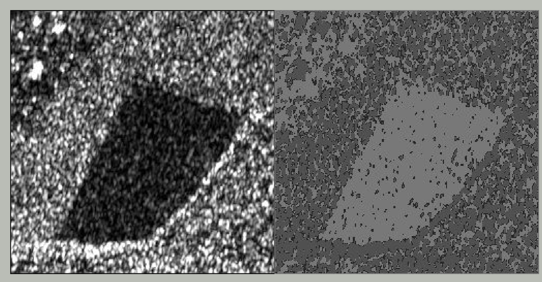
(f) One iteration and GMLC



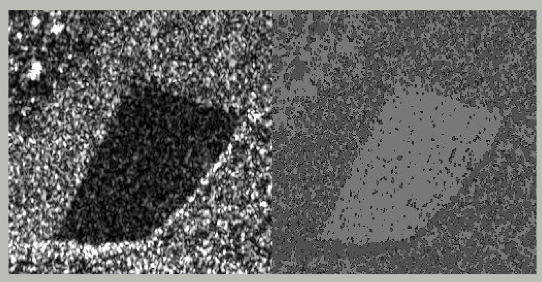
(g) **95** iterations and GMLC



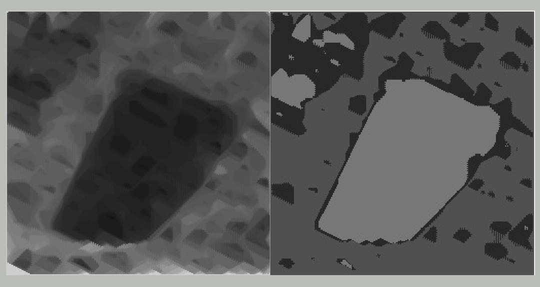
(h) Image, samples and GMLC



(i) Frost and GMLC



(j) Lee and GMLC



(k) Stack Filter **20** and GMLC

Filter	R_1/R_1	R_2/R_2	R_3/R_3
None	13.40	48.16	88.90
Sample Stack 1	9.38	65.00	93.19
Sample Stack 22	**63.52**	**74.87**	**96.5**
Stack 1	14.35	64.65	90.86
Stack 40	62.81	89.09	94.11
Stack 95	**63.01**	**93.20**	**94.04**
Frost	16.55	55.54	90.17
Lee	16.38	52.72	89.21