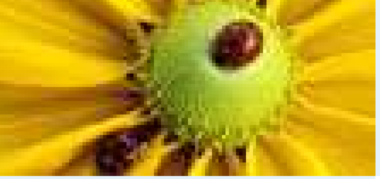
On-line and Universal Stochastic Optimization

Stefano Leonardi

"Sapienza" University of Rome



Outline

- Introduction
- Universal Stochastic Set Cover
  (Grandoni, Gupta, Miettinen, Leonardi, Sankowski & Singh, FOCS 2008)
- Conclusions

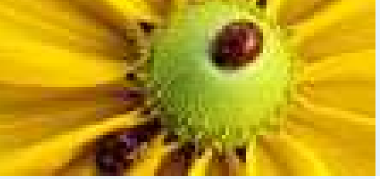
● Outline

Introduction
● On-line algorithms
● On-line Stochastic Optimization

Universal Stochastic Set Cover

Conclusions

Introduction



On-line algorithms

- On-line algorithms build incrementally a solution for an optimization problem while the input sequence is revealed step by step.
- Competitive analysis compares the performance of an on-line algorithm $\mathcal{A}$ that knows nothing about the future against a clairvoyant adversary that builds for an input instance ω an optimal solution $\texttt{opt}(\omega)$.
- Given an online algorithm $\mathcal{A}$ (possibly a randomized one), the competitive ratio is defined as

$$\max_{\omega} \frac{\mathbf{E}_r[\mathcal{A}(\omega, r)]}{\texttt{opt}(\omega)},$$

  where r is the set of random coins flipped by the algorithm.
- The results about the on-line model are in some cases unduly pessimistic.

On-line Stochastic Optimization

- Outline

Introduction
- On-line algorithms
- On-line Stochastic Optimization

Universal Stochastic Set Cover

Conclusions

- Several attempts to weaken the notion of competitive analysis by having the input sequence drawn from a probability distribution
- Several results on Paging, Scheduling, data structures: the algorithm is often oblivious to the distribution.
- We rather consider on-line algorithms for stochastic covering problems: Steiner Tree, Set cover and Facility location
- We show that the ability to sample from the distribution can be exploited to achieve much better on-line results

Outline

Introduction

Universal Stochastic Set Cover
- Universal Approximation
- Universal Set Cover
- Worst case lower bound
- Universal Stochastic Approximation
- Online Stochastic Set Cover
- Unweighted Set Cover - Length oblivious
- Related Results
- Universal Algorihm
- Universal Algorithm
- Universal Algorithm
- Weighted Set Cover - Length oblivious
- Weighted Set Cover - Length aware
- Universal Mapping for Weighted Set Cover
- Type I sets: best ratio sets
- Type II Sets: min-cost sets
- Type II Sets: min-cost sets
- Proof of Lemma 10
- Nonmetric Facility Location

Conclusions

Universal Stochastic Set Cover

Universal Approximation

- Outline

Introduction

Universal Stochastic Set Cover
- Universal Approximation
- Universal Set Cover
- Worst case lower bound
- Universal Stochastic Approximation
- Online Stochastic Set Cover
- Unweighted Set Cover - Length oblivious
- Related Results
- Universal Algorihm
- Universal Algorithm
- Universal Algorithm
- Weighted Set Cover - Length oblivious
- Weighted Set Cover - Length aware
- Universal Mapping for Weighted Set Cover
- Type I sets: best ratio sets
- Type II Sets: min-cost sets
- Type II Sets: min-cost sets
- Proof of Lemma 10
- Nonmetric Facility Location

Conclusions

- On-line algorithms are asked to take decisions while the input sequence is revealed.
- Universal algorithms [Jia et al. STOC '05] are asked to take a-priori decisions oblivious to the input sequence.
- We a-priori map each element from a universe of requests $e \in U$ to a solution $\mathbf{s}(e)$ that satisfies it.

  The solution to a sequence ω is given as

$$\mathbf{s}(\omega) = \cup_{e \in \omega} \mathbf{s}(e).$$

  The aim is finding a mapping such that the cost of $\mathbf{s}(\omega)$ is as close as possible to the optimal cost for ω, i.e., to minimize

$$\max_{\omega \subseteq U} \frac{c(\mathbf{s}(\omega))}{c(\mathrm{OPT}(\omega))}$$

Universal Set Cover

- Outline

Introduction

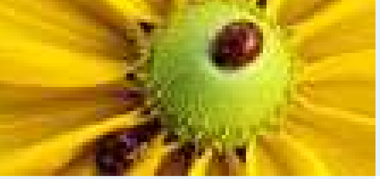
Universal Stochastic Set Cover
- Universal Approximation
- Universal Set Cover
- Worst case lower bound
- Universal Stochastic Approximation
- Online Stochastic Set Cover
- Unweighted Set Cover - Length oblivious
- Related Results
- Universal Algorihm
- Universal Algorithm
- Universal Algorithm
- Weighted Set Cover - Length oblivious
- Weighted Set Cover - Length aware
- Universal Mapping for Weighted Set Cover
- Type I sets: best ratio sets
- Type II Sets: min-cost sets
- Type II Sets: min-cost sets
- Proof of Lemma 10
- Nonmetric Facility Location

Conclusions

- We are given a universe U of n elements,
- A collection $\mathscr{S}$ of m subsets of U and a cost function $c : \mathscr{S} \rightarrow \Re_{\geq 0}$

The aim is finding an a-priori mapping $\mathbf{s} : U \rightarrow \mathscr{S}$ that minimizes:

$$\max_{\omega \subseteq U} \frac{c(\mathbf{s}(\omega))}{c(\mathsf{OPT}(\omega))}$$



Worst case lower bound

- Outline

Introduction

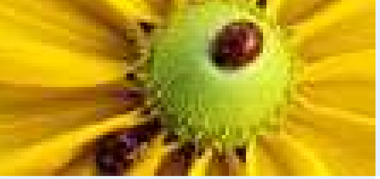
Universal Stochastic Set Cover
- Universal Approximation
- Universal Set Cover
- Worst case lower bound
- Universal Stochastic Approximation
- Online Stochastic Set Cover
- Unweighted Set Cover - Length oblivious
- Related Results
- Universal Algorihm
- Universal Algorithm
- Universal Algorithm
- Weighted Set Cover - Length oblivious
- Weighted Set Cover - Length aware
- Universal Mapping for Weighted Set Cover
- Type I sets: best ratio sets
- Type II Sets: min-cost sets
- Type II Sets: min-cost sets
- Proof of Lemma 10
- Nonmetric Facility Location

Conclusions

Theorem 1 *Any universal mapping for unweighted set cover is* $\Omega(\sqrt{n})$ *approximate against a worst case adversary.*

- Consider a set system formed by all subsets of $\sqrt{n}$ elements
- There exists a set of $\sqrt{n}$ elements that are mapped to $\sqrt{n}$ different sets
- The optimum uses only one set.

Observe that the bound is only logarithmic in the number of sets.



Universal Stochastic Approximation

- Outline

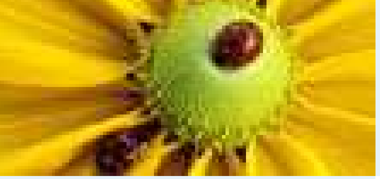
Introduction

Universal Stochastic Set Cover
- Universal Approximation
- Universal Set Cover
- Worst case lower bound
- Universal Stochastic Approximation
- Online Stochastic Set Cover
- Unweighted Set Cover - Length oblivious
- Related Results
- Universal Algorihm
- Universal Algorithm
- Universal Algorithm
- Weighted Set Cover - Length oblivious
- Weighted Set Cover - Length aware
- Universal Mapping for Weighted Set Cover
- Type I sets: best ratio sets
- Type II Sets: min-cost sets
- Type II Sets: min-cost sets
- Proof of Lemma 10
- Nonmetric Facility Location

Conclusions

In universal stochastic problems elements of U are taken to ω independently according to distribution π.

$$\mathsf{USApx}(\mathbf{S}) = \frac{\mathbf{E}_{\omega}[c(\mathbf{S}(\omega))]}{\mathbf{E}_{\omega}[c(\mathsf{OPT}(\omega))]}$$



Online Stochastic Set Cover

- Outline

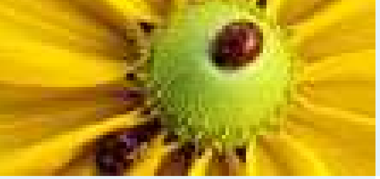
Introduction

Universal Stochastic Set Cover
- Universal Approximation
- Universal Set Cover
- Worst case lower bound
- Universal Stochastic Approximation
- Online Stochastic Set Cover
- Unweighted Set Cover - Length oblivious
- Related Results
- Universal Algorihm
- Universal Algorithm
- Universal Algorithm
- Weighted Set Cover - Length oblivious
- Weighted Set Cover - Length aware
- Universal Mapping for Weighted Set Cover
- Type I sets: best ratio sets
- Type II Sets: min-cost sets
- Type II Sets: min-cost sets
- Proof of Lemma 10
- Nonmetric Facility Location

Conclusions

In the set cover problem:

- we are given a universe U of n elements,
- and a weighted collection $\mathscr{S}$ of m subsets of U,
- the requests are elements $e_1, e_2, \ldots$ from U that need to be covered by $S \in \mathscr{S}$,
- the decisions are irrevocable.
- e_i is covered by **s**(e_i) only if not covered by the sets already picked for $e_1, \ldots, e_{i-1}$



Unweighted Set Cover - Length oblivious

- Outline

Introduction

Universal Stochastic Set Cover
- Universal Approximation
- Universal Set Cover
- Worst case lower bound
- Universal Stochastic Approximation
- Online Stochastic Set Cover
- Unweighted Set Cover - Length oblivious
- Related Results
- Universal Algorihm
- Universal Algorithm
- Universal Algorithm
- Weighted Set Cover - Length oblivious
- Weighted Set Cover - Length aware
- Universal Mapping for Weighted Set Cover
- Type I sets: best ratio sets
- Type II Sets: min-cost sets
- Type II Sets: min-cost sets
- Proof of Lemma 10
- Nonmetric Facility Location

Conclusions

Let us consider the case of $c(S) = 1$ for each $S \in \mathscr{S}$.

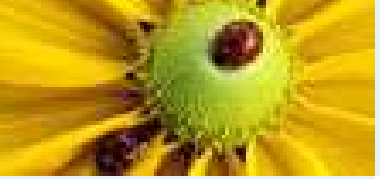
We use the standard Greedy algorithm to generate the universal mapping.

Algorithm 1: Universal mapping for unweighted set cover.

Data: Set system $(U, \mathscr{S})$.

while $U \neq \emptyset$ **do**

- let $S \leftarrow$ set in $\mathscr{S}$ maximizing $|S \cap U|$;
- **s**$(v) \leftarrow S$ for each $v \in S \cap U$;
- $U \leftarrow U \setminus S$;



Related Results

- Outline

Introduction

Universal Stochastic Set Cover
- Universal Approximation
- Universal Set Cover
- Worst case lower bound
- Universal Stochastic Approximation
- Online Stochastic Set Cover
- Unweighted Set Cover - Length oblivious
- Related Results
- Universal Algorihm
- Universal Algorithm
- Universal Algorithm
- Weighted Set Cover - Length oblivious
- Weighted Set Cover - Length aware
- Universal Mapping for Weighted Set Cover
- Type I sets: best ratio sets
- Type II Sets: min-cost sets
- Type II Sets: min-cost sets
- Proof of Lemma 10
- Nonmetric Facility Location

Conclusions

Theorem 2 *The mapping* **s** *is a length-oblivious universal mapping to the unweighted universal stochastic set cover problem with* $\mathrm{SApx} = O(\log mn)$.

In the universal case the best solution is $\tilde{\Theta}(\sqrt{n})$ approximate (Jia et al. STOC '05).

In the online case the best solution is $\tilde{\Theta}(\log n \log m)$ approximate (Alon et al. STOC '03).

Universal Algorihm

- Outline

Introduction

Universal Stochastic Set Cover
- Universal Approximation
- Universal Set Cover
- Worst case lower bound
- Universal Stochastic Approximation
- Online Stochastic Set Cover
- Unweighted Set Cover - Length oblivious
- Related Results
- Universal Algorihm
- Universal Algorithm
- Universal Algorithm
- Weighted Set Cover - Length oblivious
- Weighted Set Cover - Length aware
- Universal Mapping for Weighted Set Cover
- Type I sets: best ratio sets
- Type II Sets: min-cost sets
- Type II Sets: min-cost sets
- Proof of Lemma 10
- Nonmetric Facility Location

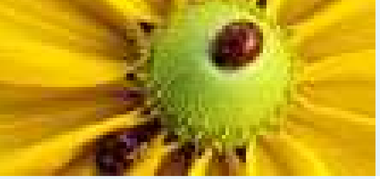
Conclusions

Fix some sequence length k and let $\mu = \mathbf{E}_{\omega \in U^k}[|\mathrm{OPT}(\omega)|]$ be the expected optimal cost.

Lemma 3 (Existence of Small (Almost-)Cover) *There exists 2μ sets in $\mathscr{S}$ which cover all but δn elements from U, for $\delta = \mu \frac{3 \ln 2m}{k}$.*

Actually Greedy needs $2\mu \log n$ sets to cover all but δn elements.

To cover the remaining elements we need $\delta n \times \frac{k}{n} = 3\mu \ln 2m$ sets.



Universal Algorithm

- Outline

Introduction

Universal Stochastic Set Cover
- Universal Approximation
- Universal Set Cover
- Worst case lower bound
- Universal Stochastic Approximation
- Online Stochastic Set Cover
- Unweighted Set Cover - Length oblivious
- Related Results
- Universal Algorihm
- Universal Algorithm
- Universal Algorithm
- Weighted Set Cover - Length oblivious
- Weighted Set Cover - Length aware
- Universal Mapping for Weighted Set Cover
- Type I sets: best ratio sets
- Type II Sets: min-cost sets
- Type II Sets: min-cost sets
- Proof of Lemma 10
- Nonmetric Facility Location

Conclusions

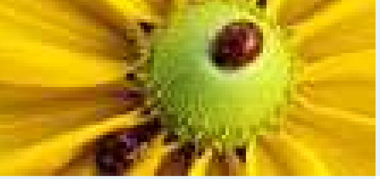
There are n^k scenarios and in at least $\frac{1}{2}$ of them the optimal cover is smaller then 2μ.

There are at most $p \leq (2m)^{2\mu} = e^{2\mu \ln 2m}$ collections C_i of 2μ sets.

Suppose for contradiction that all C_i's cover less then $n(1-\delta) < ne^{-\delta} = e^{\ln n - \delta}$ elements.

For each scenario elements can be picked from some collection C_i, so

$$\sum_{i=0}^{p} |C_i|^k \geq \frac{1}{2} n^k.$$



Universal Algorithm

- Outline

Introduction

Universal Stochastic Set Cover
- Universal Approximation
- Universal Set Cover
- Worst case lower bound
- Universal Stochastic Approximation
- Online Stochastic Set Cover
- Unweighted Set Cover - Length oblivious
- Related Results
- Universal Algorihm
- Universal Algorithm
- Universal Algorithm
- Weighted Set Cover - Length oblivious
- Weighted Set Cover - Length aware
- Universal Mapping for Weighted Set Cover
- Type I sets: best ratio sets
- Type II Sets: min-cost sets
- Type II Sets: min-cost sets
- Proof of Lemma 10
- Nonmetric Facility Location

Conclusions

$$\sum_{i=0}^{p} |C_i|^k \geq \frac{1}{2} n^k.$$

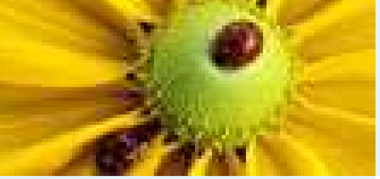
Hence, we get

$$e^{2\mu \ln 2m} \left(e^{\ln n - \delta} \right)^k > \frac{1}{2} e^{k \ln n},$$

$$e^{2\mu \ln 2m - \delta k} > \frac{1}{2} \quad \rightarrow \quad e^{-\mu \ln 2m} > \frac{1}{2}$$

But $m \geq 1$ and $\mu \geq 1$, so we get a contradiction

$$\frac{1}{2} = e^{-\ln 2} \geq e^{-\mu \ln 2m}.$$



Weighted Set Cover - Length oblivious

- Outline

Introduction

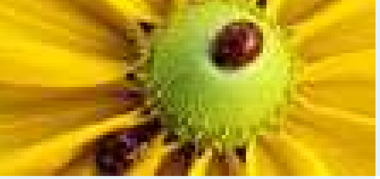
Universal Stochastic Set Cover
- Universal Approximation
- Universal Set Cover
- Worst case lower bound
- Universal Stochastic Approximation
- Online Stochastic Set Cover
- Unweighted Set Cover - Length oblivious
- Related Results
- Universal Algorihm
- Universal Algorithm
- Universal Algorithm
- Weighted Set Cover - Length oblivious
- Weighted Set Cover - Length aware
- Universal Mapping for Weighted Set Cover
- Type I sets: best ratio sets
- Type II Sets: min-cost sets
- Type II Sets: min-cost sets
- Proof of Lemma 10
- Nonmetric Facility Location

Conclusions

Theorem 4 *Any length-oblivious mapping for weighted set-cover is $\Omega(\sqrt{n})$ approximate.*

- Consider n singleton sets S_i of cost 1 and one set S_{all} of costs $\sqrt{n}$ containing all n elements
- If the algorithm maps more than $n/2$ elements to S_{all} then $k = 1$. The algorithm pays $\sqrt{n}$ in expectation whereas optimum pays 1.
- If the algorithm maps more than $n/2$ elements to sets of cost 1 then $k = n$. The algorithm pays $n/2$ in expectation whereas optimum pays $\sqrt{n}$.

A good mapping has to alternate between best ratio sets and min-cost sets



Weighted Set Cover - Length aware

- Outline

Introduction

Universal Stochastic Set Cover
- Universal Approximation
- Universal Set Cover
- Worst case lower bound
- Universal Stochastic Approximation
- Online Stochastic Set Cover
- Unweighted Set Cover - Length oblivious
- Related Results
- Universal Algorihm
- Universal Algorithm
- Universal Algorithm
- Weighted Set Cover - Length oblivious
- Weighted Set Cover - Length aware
- Universal Mapping for Weighted Set Cover
- Type I sets: best ratio sets
- Type II Sets: min-cost sets
- Type II Sets: min-cost sets
- Proof of Lemma 10
- Nonmetric Facility Location

Conclusions

Theorem 5 *There exists a length-aware mapping to the universal stochastic set cover problem with* $\mathsf{SApx} = O(\log mn)$.

Theorem 6 *There exists a length-oblivious algorithm to the online stochastic set cover problem with* $\mathsf{SApx} = O(\log mn)$.

Universal Mapping for Weighted Set Cover

- Outline

Introduction

Universal Stochastic Set Cover
- Universal Approximation
- Universal Set Cover
- Worst case lower bound
- Universal Stochastic Approximation
- Online Stochastic Set Cover
- Unweighted Set Cover - Length oblivious
- Related Results
- Universal Algorihm
- Universal Algorithm
- Universal Algorithm
- Weighted Set Cover - Length oblivious
- Weighted Set Cover - Length aware
- Universal Mapping for Weighted Set Cover
- Type I sets: best ratio sets
- Type II Sets: min-cost sets
- Type II Sets: min-cost sets
- Proof of Lemma 10
- Nonmetric Facility Location

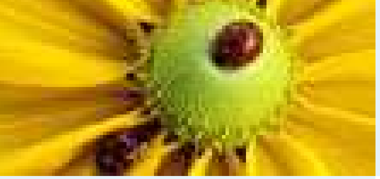
Conclusions

$\mathbf{E}[c(\text{OPT})]$ given (Actually k is sufficient.)

Data: Set system $(U, \mathscr{S})$, $c\colon \mathscr{S} \to \Re_{\geq 0}$, and $\mathbf{E}[c(\text{OPT})]$.

while $U \neq \emptyset$ **do**

- let $S \leftarrow$ set in $\mathscr{S}$ minimizing $\frac{c(S)}{|S \cap U|}$;
- if $\frac{c(S)}{|S \cap U|} > \frac{64\mathbf{E}[c(\text{OPT})]}{|U|}$ then let $S \leftarrow$ set in $\mathscr{S}$ minimizing $c(S)$;
- $\mathbf{S}(u) \leftarrow S$ for each $u \in S \cap U$;
- $U \leftarrow U \setminus S$ and $\mathscr{S} \leftarrow$ all sets covering at least one element remaining in U ;



Outline

Introduction

Universal Stochastic Set Cover
Universal Approximation
Universal Set Cover
Worst case lower bound
Universal Stochastic Approximation
Online Stochastic Set Cover
Unweighted Set Cover - Length oblivious
Related Results
Universal Algorihm
Universal Algorithm
Universal Algorithm
Weighted Set Cover - Length oblivious
Weighted Set Cover - Length aware
Universal Mapping for Weighted Set Cover
Type I sets: best ratio sets
Type II Sets: min-cost sets
Type II Sets: min-cost sets
Proof of Lemma 10
Nonmetric Facility Location

Conclusions

Type I sets: best ratio sets

Lemma 7 (Type I Set Cost) *The cost of Type I sets is* $O(\log n) \cdot \mathbf{E}[c(\mathit{OPT})]$.

- Let $S_1, \ldots, S_h$ be the Type I sets.
- Let U_i denote the set of uncovered elements just before S_i was picked.
- Since the algorithm picked a Type I set, $c(S_i) \leq |S_i \cap U_i| \frac{64\ \mathbf{E}[c(\text{OPT})]}{|U_i|}$.
- Total cost of the sets S_i bounded by

$$\sum_{i=1}^{h} c(S_i) \leq \sum_{i=1}^{h} \frac{64|S_i \cap U_i| \times \mathbf{E}[c(\text{OPT})]}{|U_i|} \leq 64\mathbf{E}[c(\text{OPT})] \sum_{t=1}^{n} \frac{1}{t}$$
$$= O(\log n)\mathbf{E}[c(\text{OPT})]$$

Type II Sets: min-cost sets

- Outline

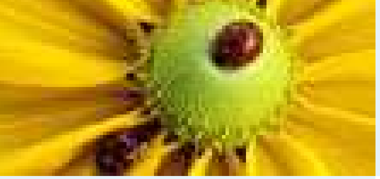
Introduction

Universal Stochastic Set Cover
- Universal Approximation
- Universal Set Cover
- Worst case lower bound
- Universal Stochastic Approximation
- Online Stochastic Set Cover
- Unweighted Set Cover - Length oblivious
- Related Results
- Universal Algorihm
- Universal Algorithm
- Universal Algorithm
- Weighted Set Cover - Length oblivious
- Weighted Set Cover - Length aware
- Universal Mapping for Weighted Set Cover
- Type I sets: best ratio sets
- Type II Sets: min-cost sets
- Type II Sets: min-cost sets
- Proof of Lemma 10
- Nonmetric Facility Location

Conclusions

- Let $S_1, \ldots, S_\ell$ be the Type II sets. $c(S_i) \leq c(S_{i+1})$
- U_i: set of uncovered elements just before S_i was picked.
- $n_i = |U_i|$ and let $k_i = n_i \frac{k}{n}$ be the expected number of elements sampled from U_i.
- Denote by $\omega_i = \omega \cap U_i$
- $\text{OPT}|_{\omega_i}$: restriction of OPT_ω to ω_i

Lemma 8 *For all* $1 \leq i \leq \ell$, $c(S_i)\mathbf{E}[|OPT|_{\omega_{i+1}}|] \leq \mathbf{E}[c(OPT|_{\omega_{i+1}})]$ *and*
$c(S_i)\left(\mathbf{E}[|OPT|_{\omega_i}|] - \mathbf{E}[|OPT|_{\omega_{i+1}}|]\right) \leq \mathbf{E}[c(OPT|_{\omega_i})] - \mathbf{E}[c(OPT|_{\omega_{i+1}})]$.



Type II Sets: min-cost sets

- Outline

Introduction

Universal Stochastic Set Cover
- Universal Approximation
- Universal Set Cover
- Worst case lower bound
- Universal Stochastic Approximation
- Online Stochastic Set Cover
- Unweighted Set Cover - Length oblivious
- Related Results
- Universal Algorihm
- Universal Algorithm
- Universal Algorithm
- Weighted Set Cover - Length oblivious
- Weighted Set Cover - Length aware
- Universal Mapping for Weighted Set Cover
- Type I sets: best ratio sets
- Type II Sets: min-cost sets
- Type II Sets: min-cost sets
- Proof of Lemma 10
- Nonmetric Facility Location

Conclusions

Lemma 9 *For every $i \in \{1, \ldots, \ell\}$, if $k_i \geq 8 \log 2n$ then $k_i \leq 16\mathbf{E}[|OPT|_{\omega_i}|] \log m$.*

The proof is similar in spirit to the one of the unweighted case but now the number of sets in the set cover is not equal to its cost.

We needed a careful restriction of the optimal solution to subproblems given by $\text{OPT}|_{\omega_i}$.

Lemma 10 (Type II Set Cost) *The expected cost of Type II sets is $O(\log mn)\,\mathbf{E}[c(OPT)]$.*

- Let j be such that $k_j \geq 8 \log 2n$ but $k_{j+1} < 8 \log 2n$.
- In expectation we see at most $8 \log 2n$ elements from U_{j+1}, with cost bounded by $8 \log 2n\,\mathbf{E}[c(\text{OPT})]$.

Proof of Lemma 10

- Outline

Introduction

Universal Stochastic Set Cover
- Universal Approximation
- Universal Set Cover
- Worst case lower bound
- Universal Stochastic Approximation
- Online Stochastic Set Cover
- Unweighted Set Cover - Length oblivious
- Related Results
- Universal Algorihm
- Universal Algorithm
- Universal Algorithm
- Weighted Set Cover - Length oblivious
- Weighted Set Cover - Length aware
- Universal Mapping for Weighted Set Cover
- Type I sets: best ratio sets
- Type II Sets: min-cost sets
- Type II Sets: min-cost sets
- Proof of Lemma 10
- Nonmetric Facility Location

Conclusions

Expected cost incurred on $S_1, \ldots, S_j$:

$$
\begin{aligned}
& \sum_{i=1}^{j} c(S_i)\mathbf{Pr}[\omega \cap (S_i \cap U_i) \neq \emptyset] \\
\leq & \sum_{i=1}^{j} c(S_i)\mathbf{E}[|\omega \cap (S_i \cap U_i)|] \\
\leq & \sum_{i=1}^{j} c(S_i)\mathbf{E}[|\omega \cap (U_i \setminus U_{i+1})|] \\
\leq & \sum_{i=1}^{j} c(S_i)\,(k_i - k_{i+1}) \leq \sum_{i=1}^{j} k_i\,(c(S_i) - c(S_{i-1})) \\
\leq & \sum_{i=1}^{j} 16\mathbf{E}[|\mathrm{OPT}|_{\omega_i}|] \log m \cdot (c(S_i) - c(S_{i-1})) \\
= & \, 16 \log m \cdot \Big(c(S_j)\mathbf{E}[|\mathrm{OPT}|_{\omega_{j+1}}|] \\
& + \sum_{i=1}^{j} c(S_i)\Big(\mathbf{E}[|\mathrm{OPT}|_{\omega_i}|] - \mathbf{E}[|\mathrm{OPT}|_{\omega_{i+1}}|]\Big)\Big) \\
\leq & \, 16 \log m \cdot \Big(\mathbf{E}[c(\mathrm{OPT}|_{\omega_{j+1}})] + \sum_{i=1}^{j} \Big(\mathbf{E}[c(\mathrm{OPT}|_{\omega_i})] - \mathbf{E}[c(\mathrm{OPT}|_{\omega_{i+1}})]\Big)\Big) \\
= & \, 16\mathbf{E}[c(\mathrm{OPT}|_{\omega_1})] \log m \leq 16\mathbf{E}[c(\mathrm{OPT})] \log m
\end{aligned}
$$

Nonmetric Facility Location

- Outline

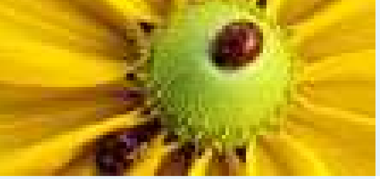
Introduction

Universal Stochastic Set Cover
- Universal Approximation
- Universal Set Cover
- Worst case lower bound
- Universal Stochastic Approximation
- Online Stochastic Set Cover
- Unweighted Set Cover - Length oblivious
- Related Results
- Universal Algorihm
- Universal Algorithm
- Universal Algorithm
- Weighted Set Cover - Length oblivious
- Weighted Set Cover - Length aware
- Universal Mapping for Weighted Set Cover
- Type I sets: best ratio sets
- Type II Sets: min-cost sets
- Type II Sets: min-cost sets
- Proof of Lemma 10
- Nonmetric Facility Location

Conclusions

The above result can be generalized to the nonmetric facility location problem:

Theorem 11 *There exists a length-aware algorithm to the online stochastic nonmetric facility location problem with* $\mathsf{SApx} = O(\log mn)$.



Outline

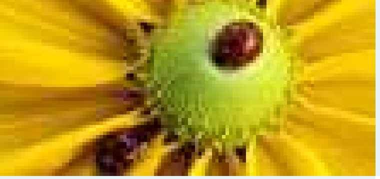
Introduction

Universal Stochastic Set Cover

Conclusions

Conclusions

Conclusions



Conclusions

The stochastic assumptions in the online setting allow to:

- beat the classical competitive $\Omega(\log n)$ lower bound for the online Steiner tree,
- beat the classical competitive $\Omega(\log n \log m)$ lower bound for the online set cover.
- beat the $\Omega(\sqrt{n})$ lower bound for universal set cover
- overcome inapproximability results for network design with outliers

Interesting problems:

- We are interested in algorithms with better expectation of ratios and length oblivious
- Develop algorithms that are risk averse,i.e., probability to deviate from the expectation for more than a ρ factor is bounded
- Tight $O(\log n)$ results for online network design with outliers